

# Electrical Heating with Horizontal Wells The Heat Transfer Problem

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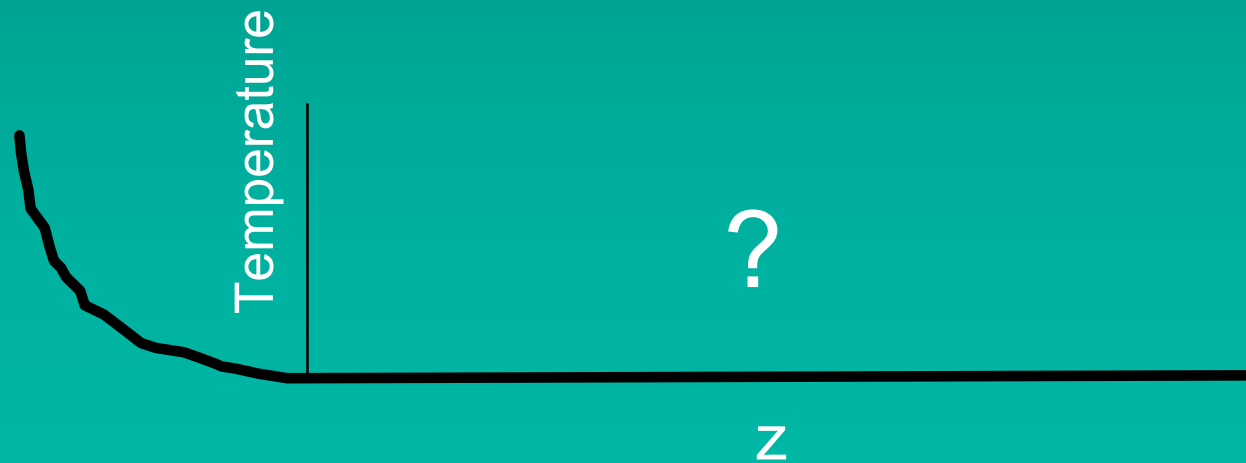
McMillan-McGee Corp.

Frederick E. Vermuelen

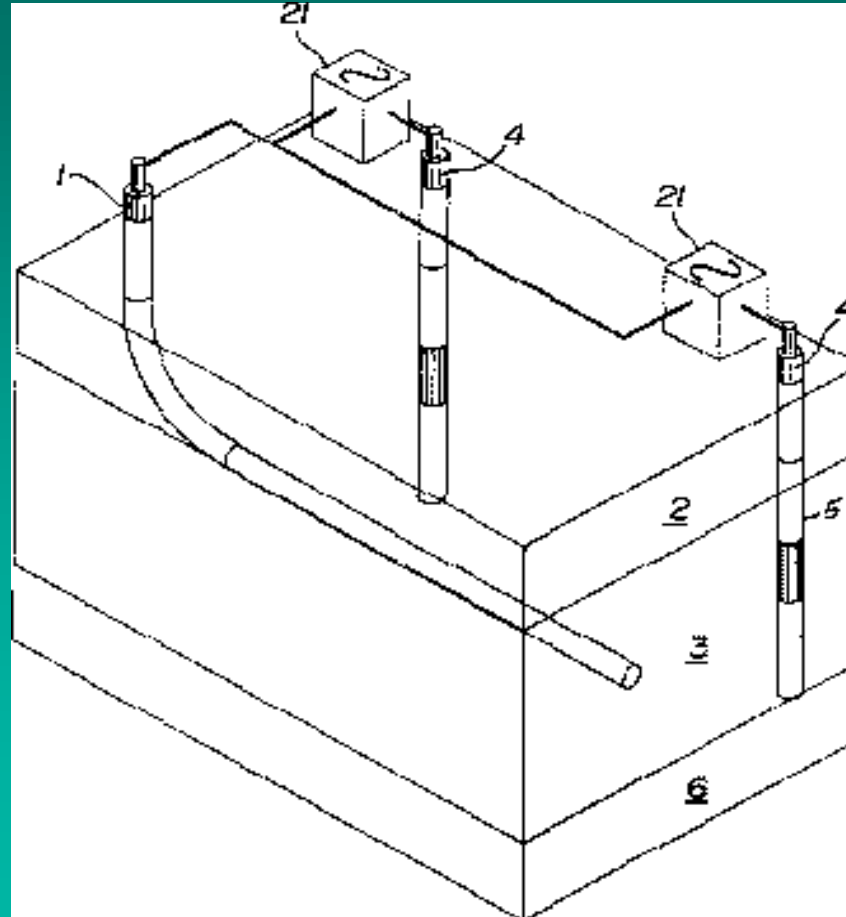
University of Alberta

# Outline

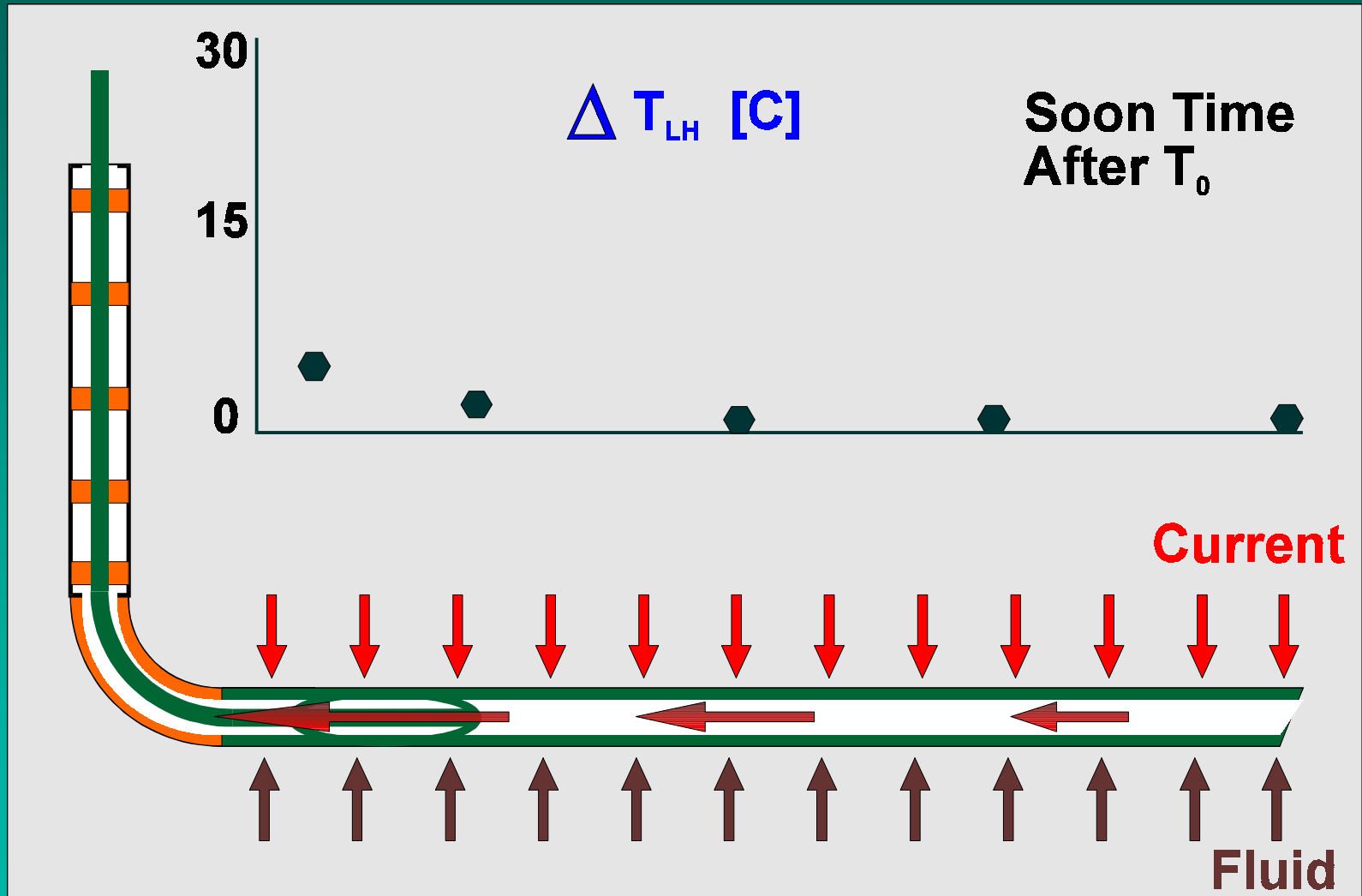
- Present our mathematical solution of the axial heat transfer problem for electrical heating of a horizontal well.
- Present some results.



# TEXCAN Horizontal Well Electrical Heating Technology



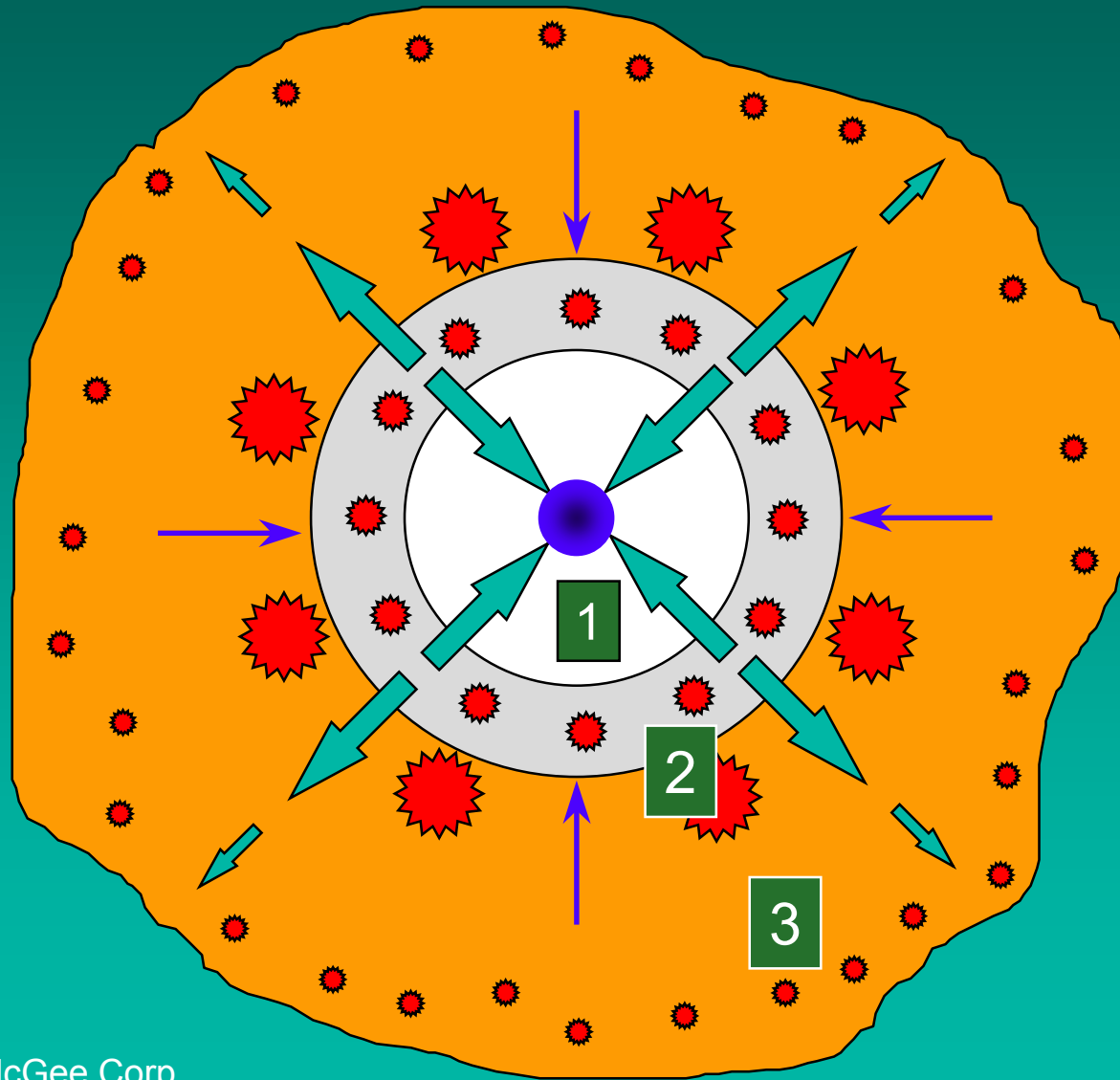
# Process Dynamics at $T_0 +$



# Calculation Procedure

- Derive the Heat Transfer PDE for each region.
- Obtain the Laplace Transforms of these equations.
- Solve the above equations.
- Construct a system of linear equations and determine the unknown coefficients using the boundary conditions and matrix algebra.
- Determine the energy transfer into wellbore and solve the transformed heat transfer equation there.
- Invert the Laplace solution into the time domain.

# Heat Transfer Mechanisms



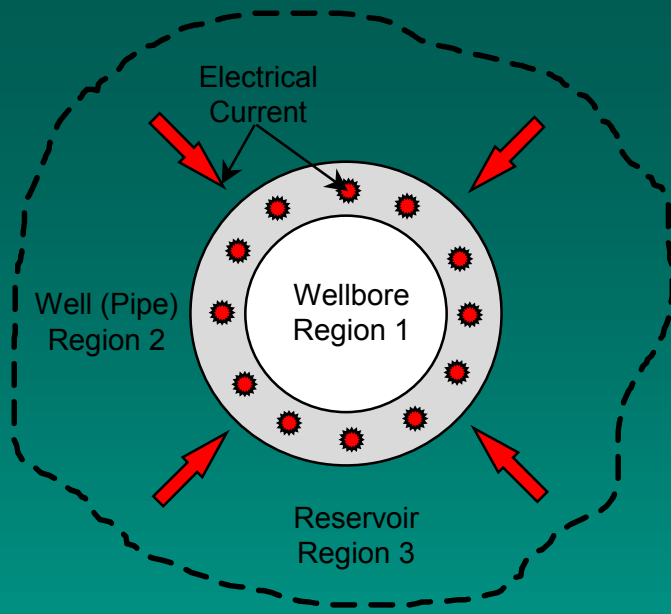
Electrical



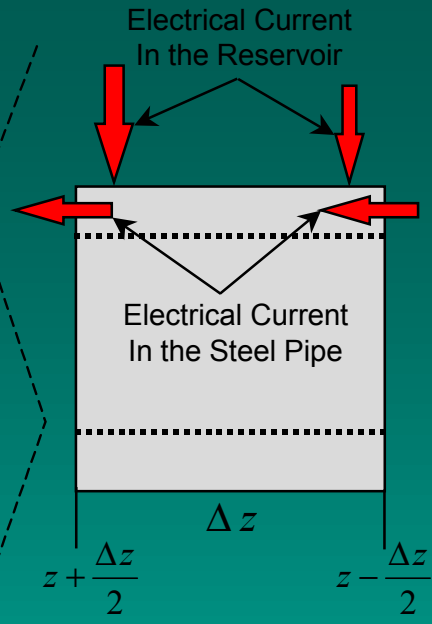
Convection



Conduction



End View

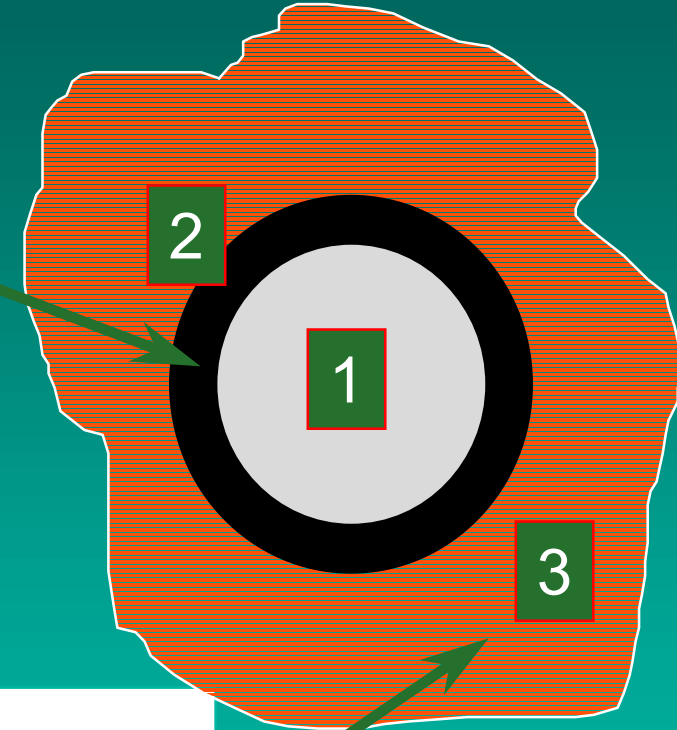


Top View

# Heat Transfer Equations

## Regions 2 and 3

$$\lambda_s \frac{\partial^2 T_2}{\partial r^2} + \frac{\lambda_s}{r} \frac{\partial T_2}{\partial r} + \dot{q}_2(z) = \rho C_s \frac{\partial T_2}{\partial t}$$

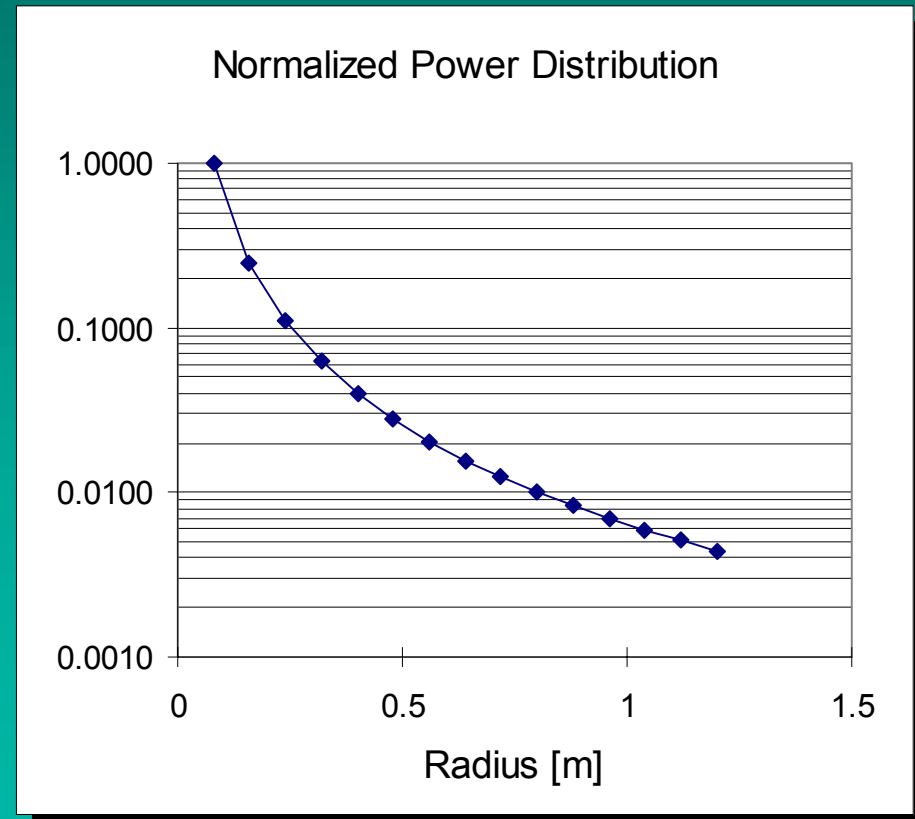


$$\lambda_r \frac{\partial^2 T_3}{\partial r^2} + \left( \lambda_r + \rho C_r \frac{q_f}{2\pi l} \right) \frac{1}{r} \frac{\partial T_3}{\partial r} + \dot{q}_3(r) = \rho C_r \frac{\partial T_3}{\partial t}$$

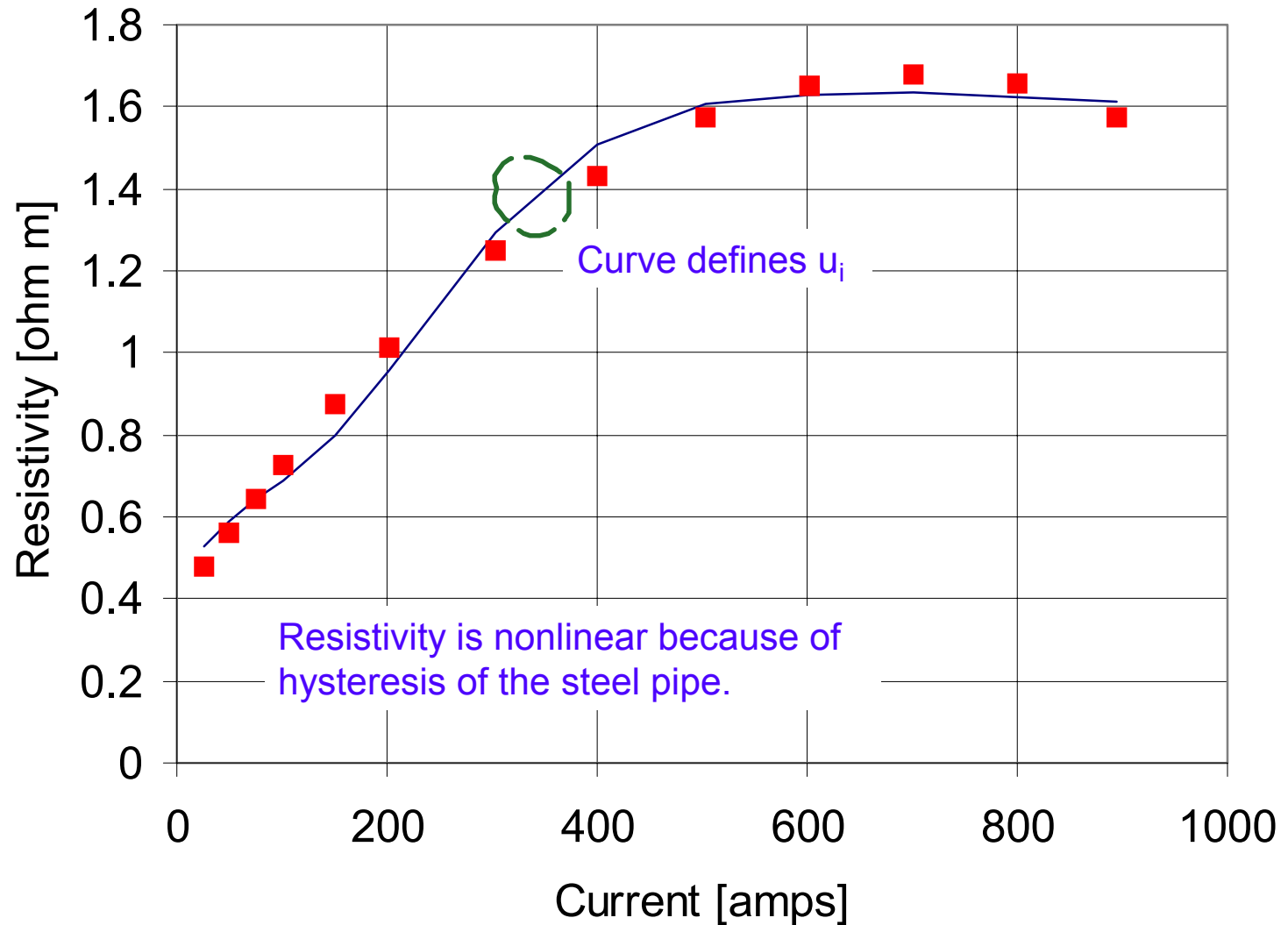
# The Energy Source In Region 3

$$\dot{q}_2(z) = \rho_r |J(r)|^2 = \rho_r \left( \frac{I_t}{2\pi r l} \right)^2$$

At a distance of 10 wellbore radii from the wellbore, the heating rate is reduced by a factor of 100!



# Resistivity as a Function of Current Fit to a 3rd Order Polynomial (data from Stroemich et. al.)



# The Energy Source In Region 2

axial accumulation of current

$$\dot{q}_2(z) = \rho_s(I(z)) \cdot \frac{I(z)^2}{A_s^2} = \rho_s(z) \cdot \frac{I_t^2}{A_s^2} \left(\frac{z}{l}\right)^2$$

nonlinear hysteresis effects

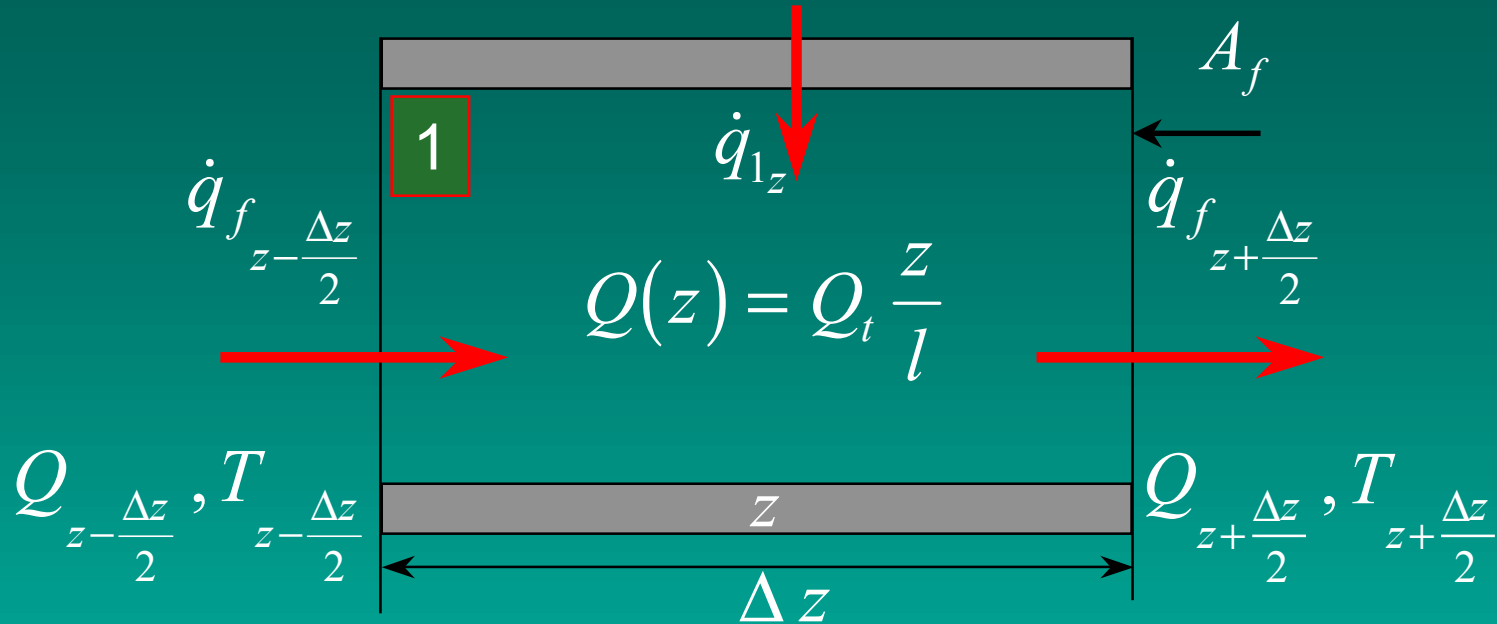
$$\rho_s(I(z)) = \rho_s(z) = \sum_{i=0}^n u_i \left(\frac{I_t}{l}\right)^i z^i$$

$I_t \left(\frac{z}{l}\right)$

$$\dot{q}_2(z) = \frac{1}{A_s^2} \sum_{i=0}^n u_i \left(\frac{I_t}{l}\right)^{i+2} z^{i+2}$$


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# Heat Transfer Equation - Region 1

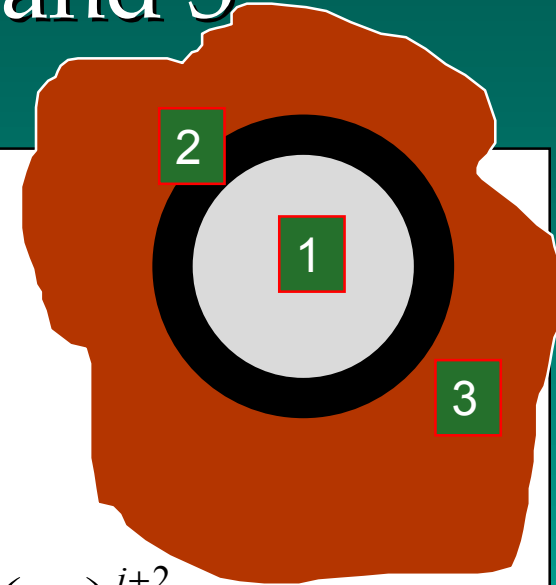


$$z \frac{\partial T_1(z)}{\partial z} = -\alpha \frac{\partial T_1(z)}{\partial t} + 2\alpha \frac{\lambda_s}{\rho C_f r_i} \frac{1}{\partial r} \frac{\partial T_2(r)}{\partial r} \Big|_{r=r_i}$$

$$\alpha = \frac{A_f l}{Q_t}$$

$\dot{q}_{1z}$  points to the term  $2\alpha \frac{\lambda_s}{\rho C_f r_i} \frac{1}{\partial r} \frac{\partial T_2(r)}{\partial r} \Big|_{r=r_i}$

# Transformed Heat Transfer Equations Regions 1, 2, and 3



1

$$z \frac{d \hat{T}_1(z)}{d z} + s \alpha \hat{T}_1 = \alpha T_0 + 2 \alpha \frac{\lambda_s}{\rho C_f} \frac{1}{r_i} \frac{d \hat{T}_2(r)}{d r} \Big|_{r=r_i}$$

2

$$\lambda_s \frac{d^2 \hat{T}_2}{d r^2} + \frac{\lambda_s}{r} \frac{d \hat{T}_2}{d r} - s \rho C_s \hat{T}_2 = -\rho C_s T_0 - \frac{1}{s A_s^2} \sum_{i=0}^n u_i \left( \frac{I_t}{l} \right)^{i+2} z^{i+2}$$

3

$$\lambda_r \frac{d^2 \hat{T}_3}{d r^2} + \left( \lambda_r + \rho C_r \frac{q_f}{2 \pi l} \right) \frac{1}{r} \frac{d \hat{T}_3}{d r} - s \rho C_r \hat{T}_3 = -\rho C_r T_0 - \frac{\rho_r}{s} \left( \frac{I_t}{2 \pi l} \right)^2 \frac{1}{r^2}$$

# Solution of Transformed Heat Transfer Equations in Regions 1 and 2

2

$$\hat{T}_2(r, z, s) = C(s) I_0 \left( r \sqrt{s \frac{\rho C_s}{\lambda_s}} \right) + D(s) K_0 \left( r \sqrt{s \frac{\rho C_s}{\lambda_s}} \right) + \frac{T_0}{s} + \sum_{i=0}^n \frac{u_i}{s^2 \beta_s \lambda_s A_s^2} \left( \frac{I_t}{l} \right)^{i+2} z^{i+2}$$

3

$$\hat{T}_3(r, s) = E(s) \frac{I_p(r \sqrt{s \beta_r})}{r^p} + F(s) \frac{K_p(r \sqrt{s \beta_r})}{r^p} - \gamma \frac{I_p(r \sqrt{s \beta_r})}{r^p} \int_{r_w}^r \frac{K_p(x \sqrt{s \beta_r})}{x^{2p-1}} dx + \gamma \frac{K_p(r \sqrt{s \beta_r})}{r^p} \int_{r_w}^r \frac{I_p(x \sqrt{s \beta_r})}{x^{2p-1}} dx$$

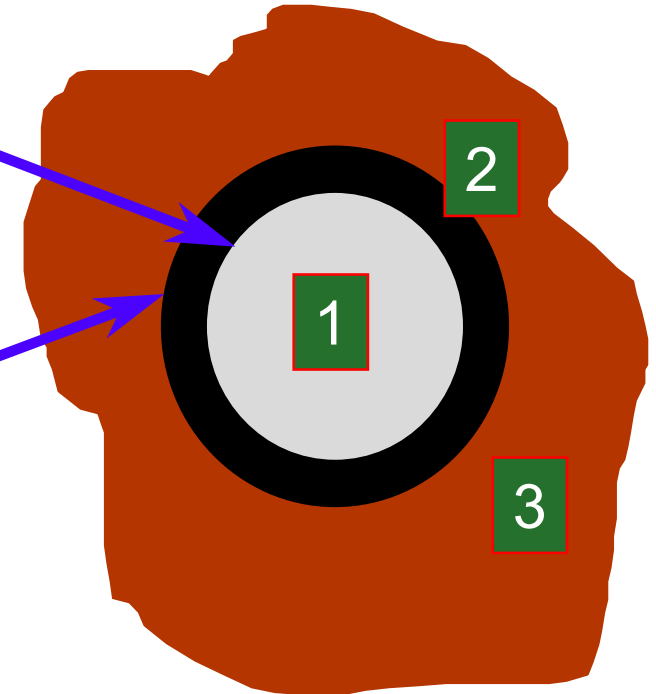
# BC's and Expressions for C(s) and D(s)

Boundary  
Conditions

$$\hat{T}_2(r_i, s) = \hat{T}_1(z, s)$$

$$\hat{T}_2(r_w, s) = \hat{T}_3(r_w, s)$$

$$\lambda_s \left. \frac{d\hat{T}_2(r, s)}{dr} \right|_{r=r_w} = \lambda_r \left. \frac{d\hat{T}_3(r, s)}{dr} \right|_{r=r_w}$$



With C(s) and D(s) known,  $T_2(r, s)$  is fully described.

$$C(s) = g_1(s) \left\{ \hat{T}_1(z, s) - \frac{T_0}{s} \right\} + g_2(s) \sum_{i=0}^n \eta_i(s) z^{i+2}$$

$$D(s) = g_3(s) \left\{ \hat{T}_1(z, s) - \frac{T_0}{s} \right\} + g_4(s) \sum_{i=0}^n \eta_i(s) z^{i+2}$$

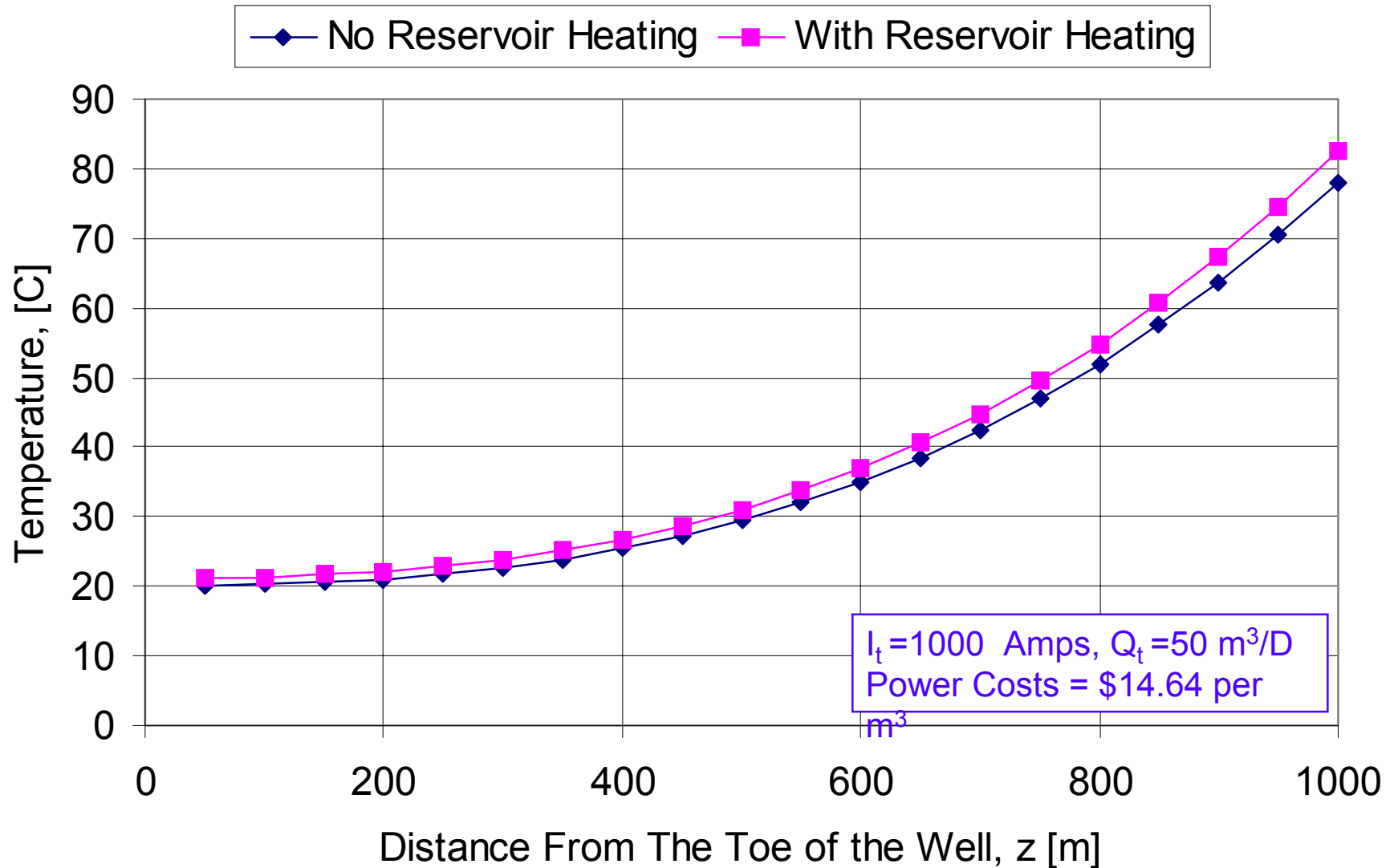
# Explicit Expression for the Energy Source in Region 1 and Final Solution of the Transformed Equation

$$2\alpha \frac{\lambda_s}{\rho C_f} \frac{1}{r_i} \frac{d\hat{T}_2}{dr} \Big|_{r=r_i} = \omega(s) \hat{T}_1(z, s) + \gamma(s) \sum_{i=0}^n \eta_i(s) z^{i+2} - \omega(s) \frac{T_0}{s}$$

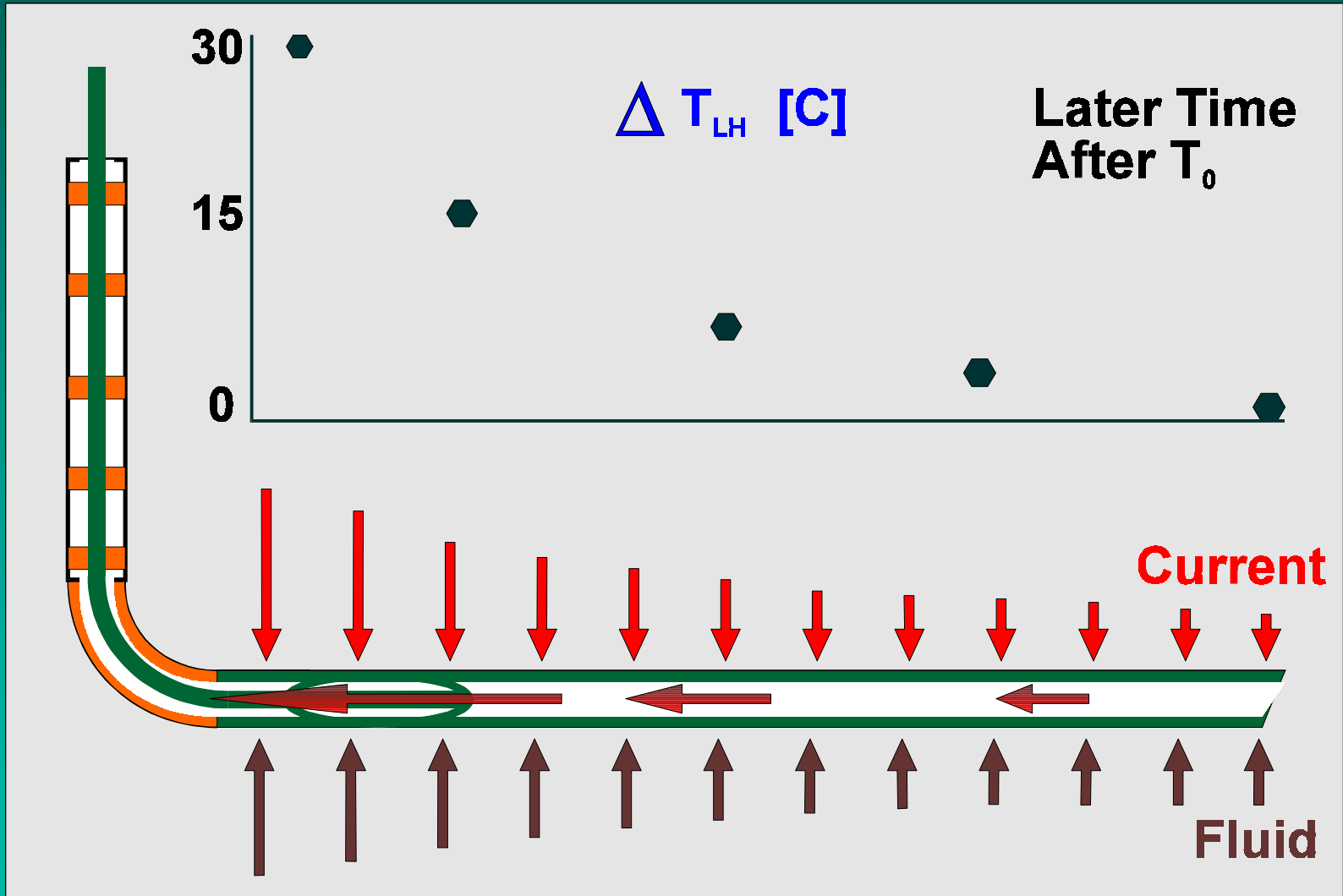
$$z \frac{d\hat{T}_1}{dz} + (s\alpha - \omega(s)) \hat{T}_1 = \frac{T_0}{s} (s\alpha - \omega(s)) + \gamma(s) \sum_{i=0}^n \eta_i(s) z^{i+2}$$

$$\hat{T}_1(z, s) = \frac{T_0}{s} + \gamma(s) \sum_{i=0}^n \frac{\eta_i(s) z^{i+2}}{i + s\alpha - \omega(s) + 2} - \left( \frac{z}{z_0} \right)^{\omega(s) - s\alpha} \gamma(s) \sum_{i=0}^n \frac{\eta_i(s) z_0^{i+2}}{i + s\alpha - \omega(s) + 2}$$

# Temperature Distribution Along the Length of a Horizontal Well



# Process Dynamics at $T_0 ++$



# Conclusions

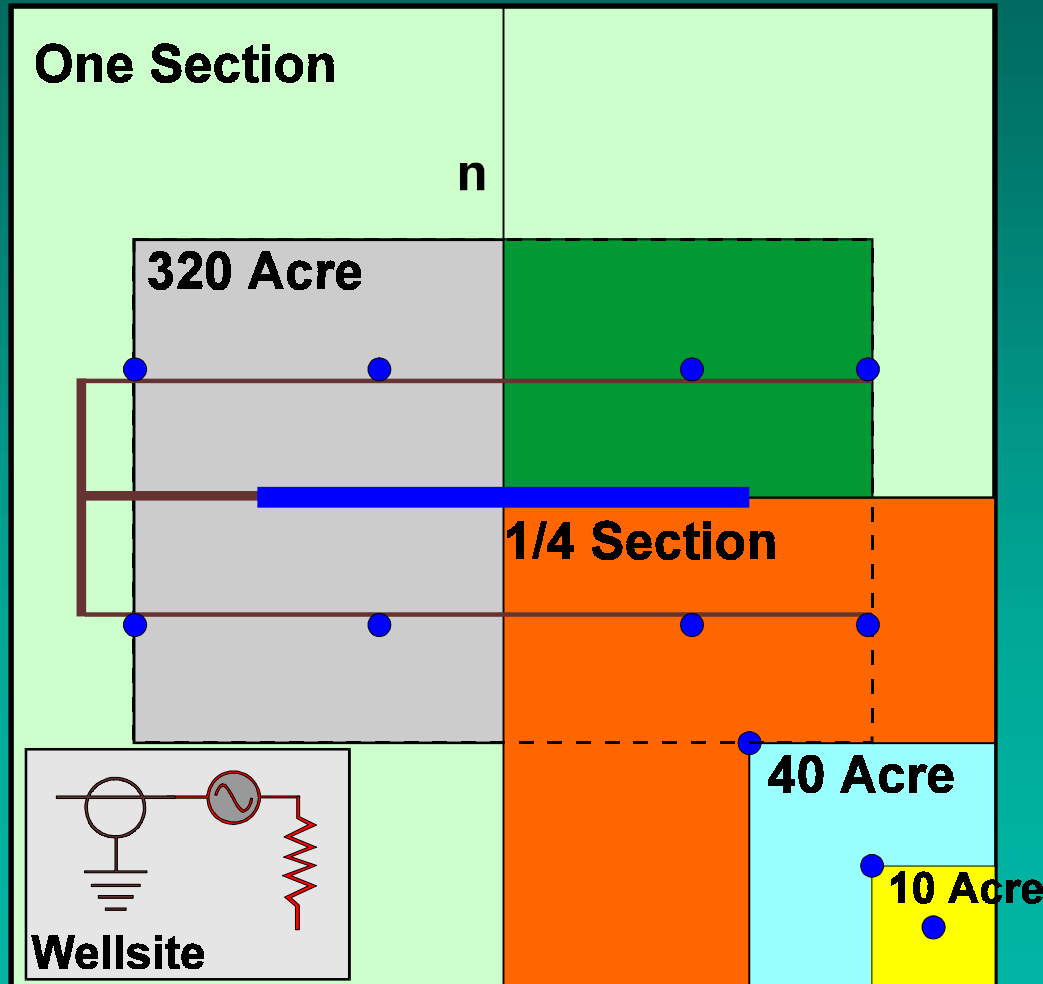
- A semi-analytical model has been developed to determine the axial temperature distribution in a horizontal well that is being electrically heated.
- The temperature distribution is not uniform as presented, however the system can be modified for a more uniform temperature distribution
- The resulting temperature distribution, under normal operating conditions is determined by the current in the steel pipe as opposed to the current in the reservoir

# Acknowledgments

An illustration of an oil well with a wooden derrick structure. Above the well, there is a large, dark red, irregular shape representing an oil spill, with several smaller red droplets falling from it. The background is a teal color with a grid pattern on the left side.

- University of Alberta
- Texaco Canada Petroleum Inc.
- Alberta Department of Energy (formally AOSTRA)

# Commercial Implementation



Horizontal Well on  
320 Acre Spacing

$$\frac{V}{H} = 8 : 1$$

Vertical Wells on  
40 Acre Spacing

Ground Return